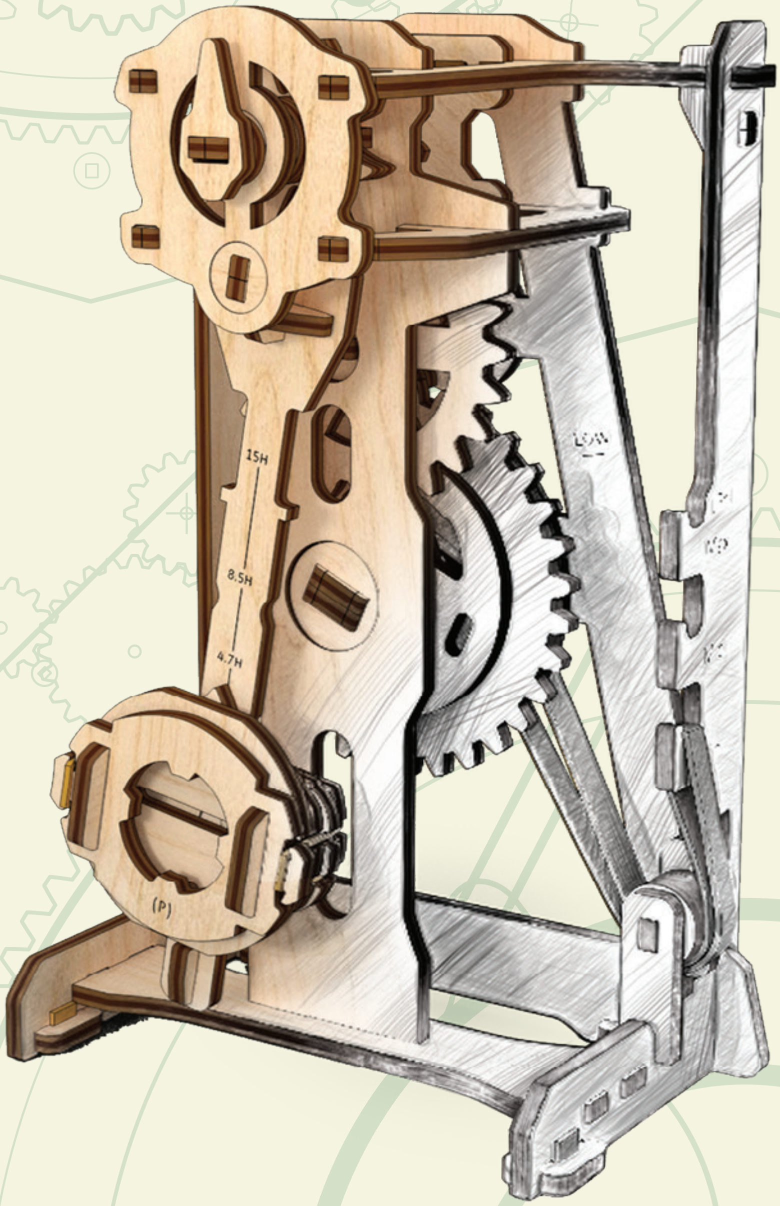
STEM LAB by UGEARS

MECHANICAL MODEL

PENDULUM



Handbook of A Young Engineer

§1 Introduction

Peter Henlein
Born in 1479, Nuremberg, Germany.
The inventor of the world's first watch.

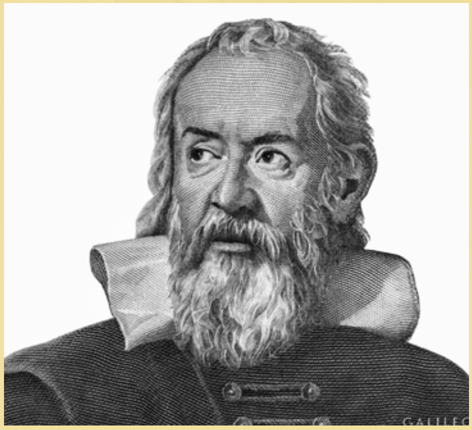
How a young lad sent the world swinging

Modern life is virtually unimaginable without clocks or watches. Certainly, one day everybody asked themselves how does an old alarm clock work? Or maybe even tried to disassemble and fix some watch. So many small screws, gears and springs inside. Take those tickers apart and – and here you are, sitting over a pile of tiny details. Which is the main one?

In the beginning of the 16th century, a locksmith from Nuremberg, Peter Henlein, created a watch with a mechanism made of gears and a spring. It was a very simple device, and its accuracy left much to be desired: the watch ran fast or slow depending on many factors, first of which was the spring value and gear tension. The watch lacked an escapement mechanism, the "heart" of a classic timepiece.

Two great minds of the 17th century gave the clock its heart, Galileo Galilei and Christiaan Huygens

Galileo Galilei
Born 15 February 1564, Pisa, Duchy of Florence
Galileo Galilei was an Italian physicist, engineer, astronomer, philosopher, and mathematician that invaluably influenced the development of many scientific disciplines. He was the first to use a telescope to observe celestial bodies and made a range of major astronomical discoveries. Galilei became a founder of an experimental physics. His experiments disproved Aristotelian geocentric view of the Earth and laid the groundwork of a classic mechanics.



Christiaan Huygens
Born 14 April 1629, The Hague, Dutch Republic
Christiaan Huygens was a Dutch engineer, physicist, mathematician, astronomer, and inventor. He became a first foreign fellow of The Royal Society of London as well as a member of French Academy of Sciences to later head it. Huygens is one of the founders of theoretical mechanics and the theory of probability

§2 Historical reference

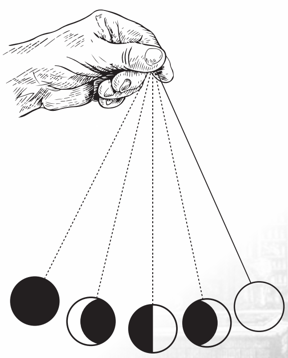
Arched windows in the building near Pisa tower (Italy)

It all started with a pendulum. It is believed that it was invented in Pisa

In 1584 as a medical student, nineteen year old Galilei had to attend services in the Pisa Cathedral.

The legend says that one of those days he was distracted by a curious phenomenon: massive bronze chandeliers attached to the ceiling of the building with long chains were swinging, being pushed by the wind and each oscillation took the same time while the amplitude* was growing smaller and smaller.

At that time, there were no accurate time measuring devices. But the young man found a way: he gauged the oscillation time against his own heart rate. Counting the number of heart beats, the experimenter came to the conclusion that while oscillations were converging and their amplitude grew shorter, the time of each oscillation remained exactly the same.



Back at home, Galilei proceeded with his experiment. He made all sorts of objects swing: a door key, pebbles on a string, an empty ink-pot, anything he could find to simulate the oscillations of the chandelier.

The outcome of his initial research came in the form of a very simple pendulum: a small bob on a string. Pulled aside and dropped, the bob would oscillate for a long while.

Inspired, the inventor immediately asked himself: how does the length of the string and the weight of the bob effect the combined oscillations time of the pendulum? Having extended the string, he figured out that the oscillations slow down. The oscillations' frequency of the pendulum with a string of 100 cm was around 2 seconds. Increased four-fold up to 400 cm, the oscillation frequency rose up to 4 seconds. Thus, four times longer string made the pendulum swing in twice as long a period. When extended times-nine, the oscillations frequency increased three-fold. At the same time, the length of the amplitude didn't have any effect on the outcomes of the experiment.

When Galilei experimented with the influence of the weight of the bob on the oscillation frequency, he got the most unexpected result. A heavy metal bob and a light cork oscillated in a perfect unison. It turned out that the weight of the bob has no effect on the frequency of the oscillations.

* Amplitude - for mechanical oscillations of the body – is the maximum value of the deviation from the equilibrium position.

The research led the inventor to draw a conclusion that the pendulum swing always take the same amount of time, independently of the amplitude that became known as the first pendulum law.

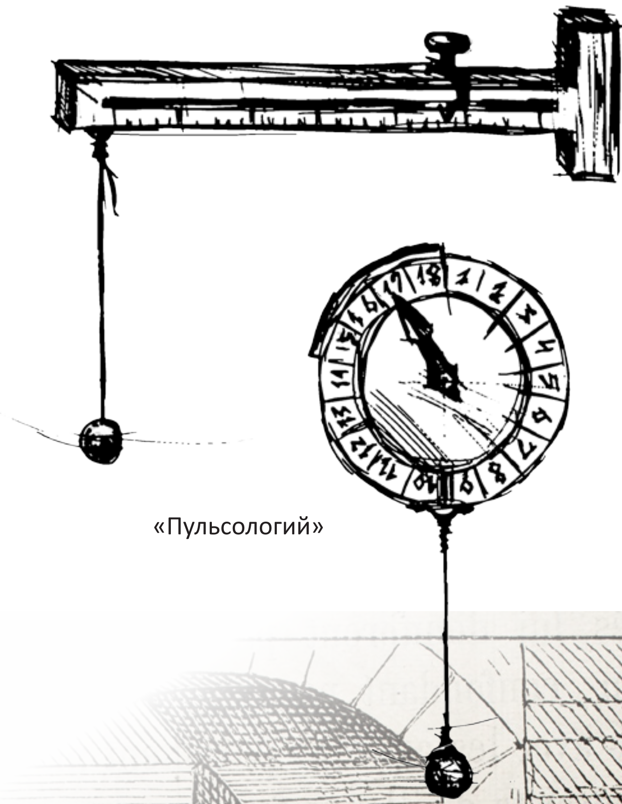
Galilei assumed that his innovation could be useful to doctors to check a patients' pulse. He connected a pendulum to a simple counter and built a device that he called "Pulsilogium".

His time measuring was still based on his own pulse

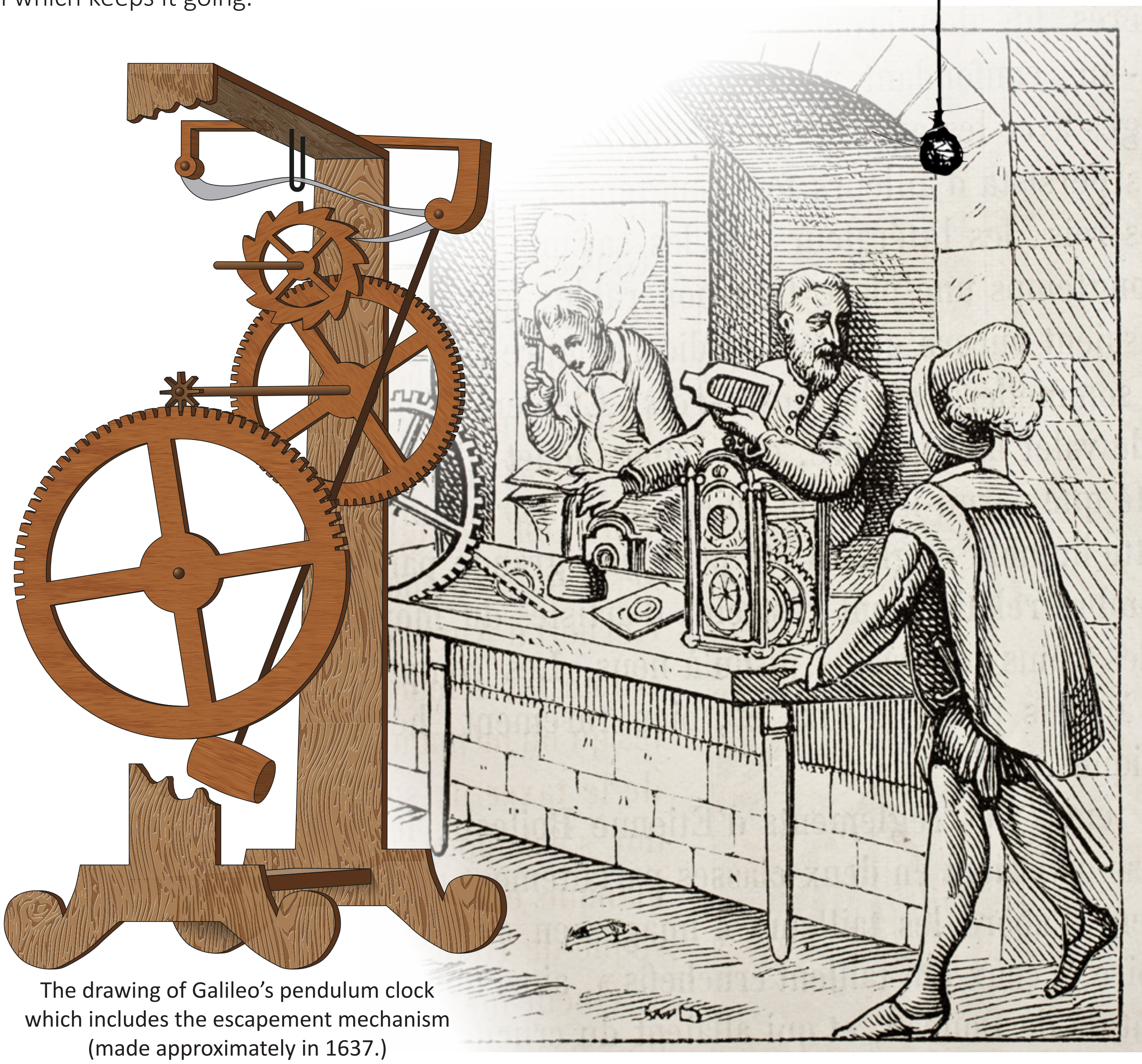
We can only assume whether a genius scientist planned to use a pendulum to create an accurate clock. Alas, he didn't come back to it until he was over 70 years old and lost his eye-sight.

According to his father's theories, Galileo's son Vincenzo along with the genius's student Viviani sketched a clock mechanism. While it is unknown whether they built an actual device, the clock mechanism created later was based on this initial design.

For his escapement mechanism Galileo used a pinwheel and a pair of curved pawls connected to a pendulum. As the pendulum swings, one pawl lifts clear of the pins allowing the wheel to rotate until 'caught' by the other pawl. As the pawl is caught, it imparts a small impulse to the pendulum which keeps it going.

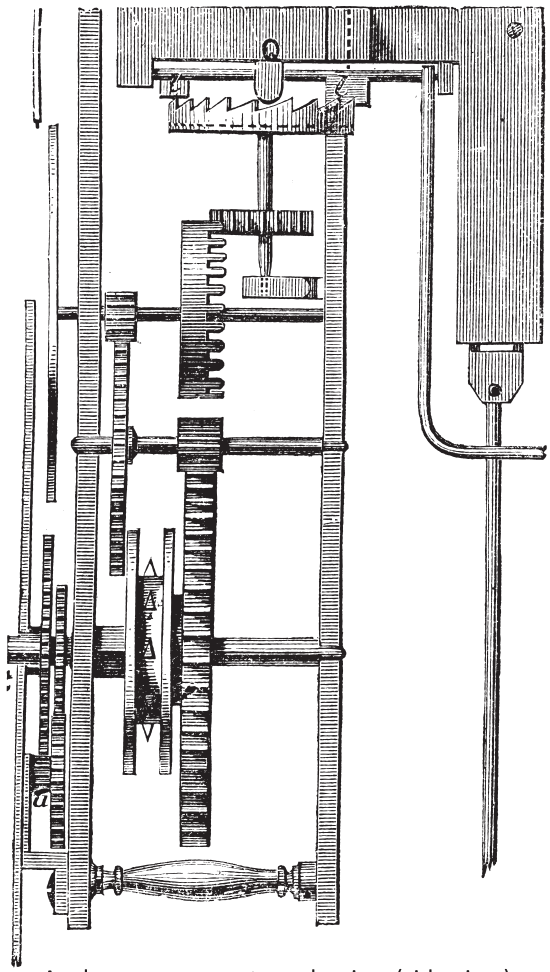


«Пульсологий»



The drawing of Galileo's pendulum clock which includes the escapement mechanism (made approximately in 1637.)

Old clock with an anchor escapement mechanism



Anchor escapement mechanism (side view)

The great and fabulous Mechanical Clock!

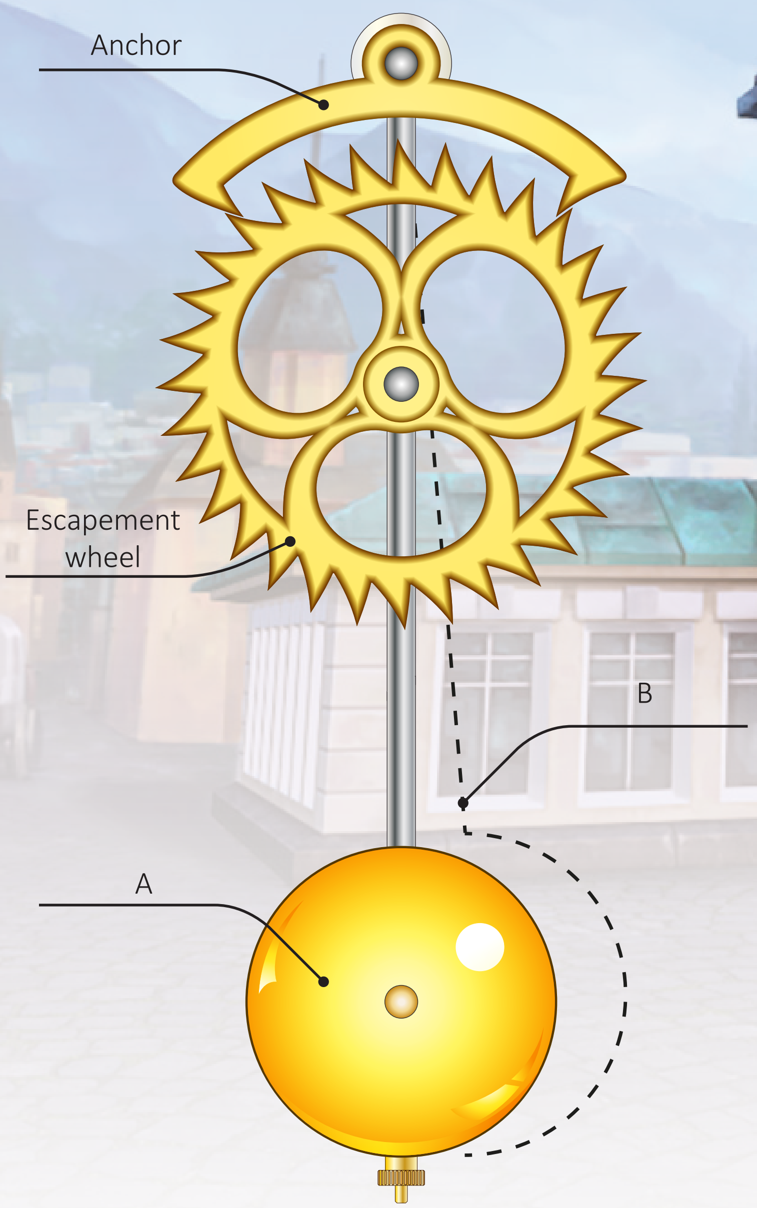
In 1657, Christiaan Huygens issued a paper describing his recent invention – a clock with a pendulum. Huygens' clocks were precise and in the next 40 years the scientist returned many times to his inventions to improve them even more and to learn more about the properties of the pendulum.

In his clock, Huygens used a spindle-type mechanism that wasn't as universal as the one suggested by Galileo. In particular, it could only work with a pendulum with a large oscillatory amplitude.

A stable pace of a clock with a large oscillatory amplitude required in the spindle-type mechanism was virtually unachievable. There had to be a way to reduce the amplitude. That way came in the form of an anchor escapement mechanism.

Escapement mechanism

The escape wheel is attached to a cylinder gear with a chain over it. The anchor, a slightly curved detail attached to the top of the pendulum, has two teeth on each of its arms that are called pallets. Swinging along with the pendulum, the anchor alternately catches and releases an escape wheel tooth with the pallets. It is believed that the credit of invention of an anchor escapement belongs to Robert Hooke and dated 1670.



Anchor escapement mechanism (front view)

The anchor allows the escape wheel to turn only one cog for each ½ pendulum period. A – pendulum anchor before locking the escape wheel and making a one-cog step. B – pendulum anchor after releasing the escape wheel and making a one-cog step).

In 1671, English clockmaker William Clement made the first clock with anchor escapement challenging the precedence of Hooke's invention.

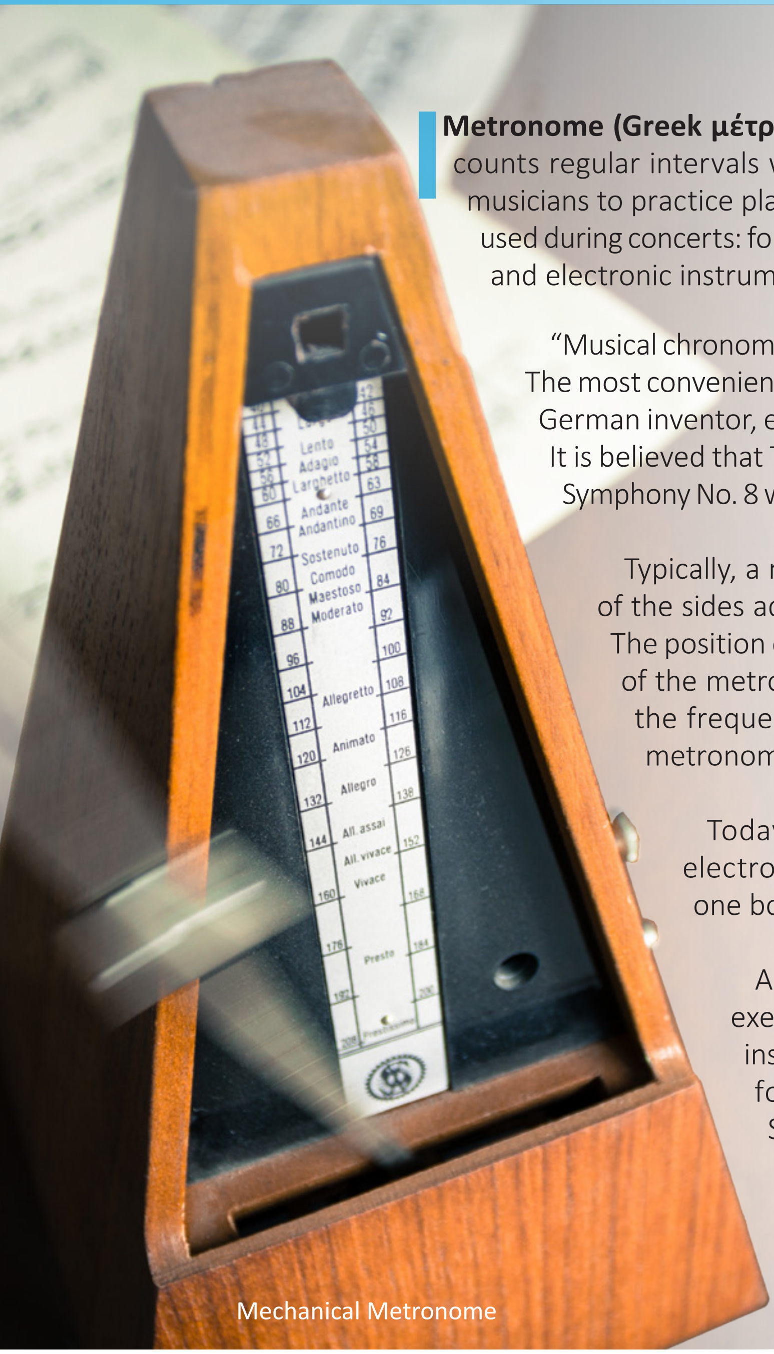
Later, in 1715, another English inventor, clockmaker and geophysicist George Graham, improved the anchor escapement mechanism to make it much more accurate, up to 0.1 sec. Graham's mechanism was used for almost 200 years up until 1890.

Naturally, George Graham was not the last inventor who had ideas about how to make clocks more accurate and efficient – many clockmakers suggested their approach to the essential mechanism. The history of clockmaking has seen over 200 anchor escapement mechanisms invented.

In the 19th century the electric watch came into the picture for the first time. The pendulum in this type of mechanism was controlled by an electric board. In the mid 20th century, the quartz watch was introduced. It uses an electronic oscillator that creates a signal with a very precise frequency and is regulated by a quartz crystal.

Today, the mechanical pendulum clock can't compete with electronic timepieces. They remain aesthetically valuable and their history is an important stage of the development of scientific thought that will inspire generations to come.

§3 About the mechanism and the range of its use



Metronome (Greek μέτρον «measure» + νέμω «I lead») is a device that counts regular intervals with clicks or other sounds. Mainly, in is used by musicians to practice playing to a steady pulse. In recent years, it was also used during concerts: for example, to synchronize pre-recorded background and electronic instruments.

“Musical chronometers” came into use in the end of the 17th century. The most convenient was the metronome created in the early 1800s by German inventor, engineer, and showman Johann Nepomuk Maelzel. It is believed that The second movement of Ludwig van Beethoven’s Symphony No. 8 was an affectionate parody of Maelzel’s Metronome.

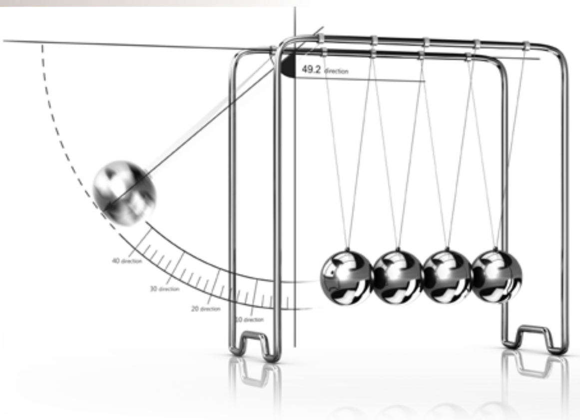
Typically, a metronome has a pyramid shaped body with one of the sides adjusted to accommodate a pendulum with a bob. The position of the bob on the pendulum effects the frequency of the metronome’s clicks. The higher the bob sits, the lower the frequency and vice versa. The scale on the face of the metronome indicates the frequency of the clicks.

Today, along with mechanical metronomes we use electronic ones. The later ones are often combined in one body with tuners.

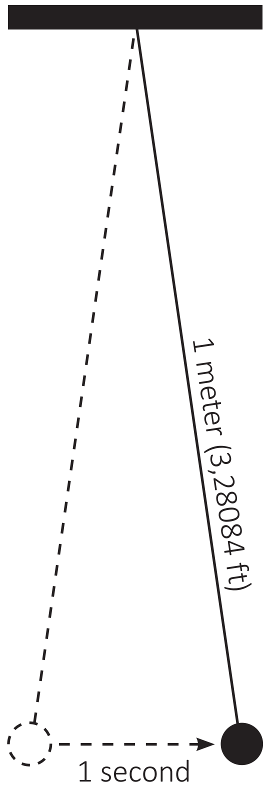
A metronome can also be used to time physical exercises, scientific experiments, or even as a musical instrument of its own – like in Poème Symphonique for 100 Metronomes by György Ligeti or Alfred Schnittke’s Dead Souls Two movements.

Mechanical Metronome

Newton’s Cradle is a mechanical system named after Isaac Newton that that demonstrates conservation of momentum and energy and the transformation of kinetic energy to potential and back. With the lack of opposite force (tension) the system could work forever, but in reality it’s impossible.



Seconds pendulum is a pendulum that oscillates in a 2 second period: one second for a swing in one direction and one second for the return swing. The oscillations frequency is ½ hertz*.



The weight is suspended from the pivot and swings freely. When displaced from its equilibrium position, the pendulum is subject to gravity, tension and elastic force.

When released, the restoring force combined with the pendulum's mass causes it to oscillate about the equilibrium position, swinging back and forth.

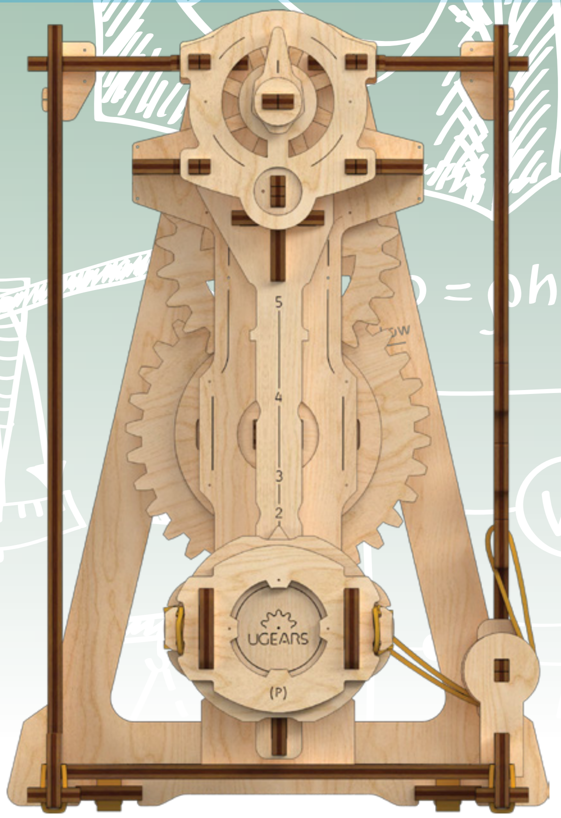
The time for one complete cycle, a left swing and a right swing, is called the period.

The period depends on the length of the pendulum as well as to some extent on the position of the weight (the moment of inertia in relation to the center of gravity), and the amplitude (the swing).

Mechanical Clocks are the clocks using a pendulum mechanism for measuring time. It is propelled by the weights, springs or electric source of energy. To measure time, they use the inertia of an oscillatory system – the pendulum, regular or spring-based that uses a spiral spring as a balance controller (+/-)

* a frequency of 1 Hz means one oscillation per second

§4 Physics and Mechanics explained in “Pendulum” STEM-model



Terms and Concepts

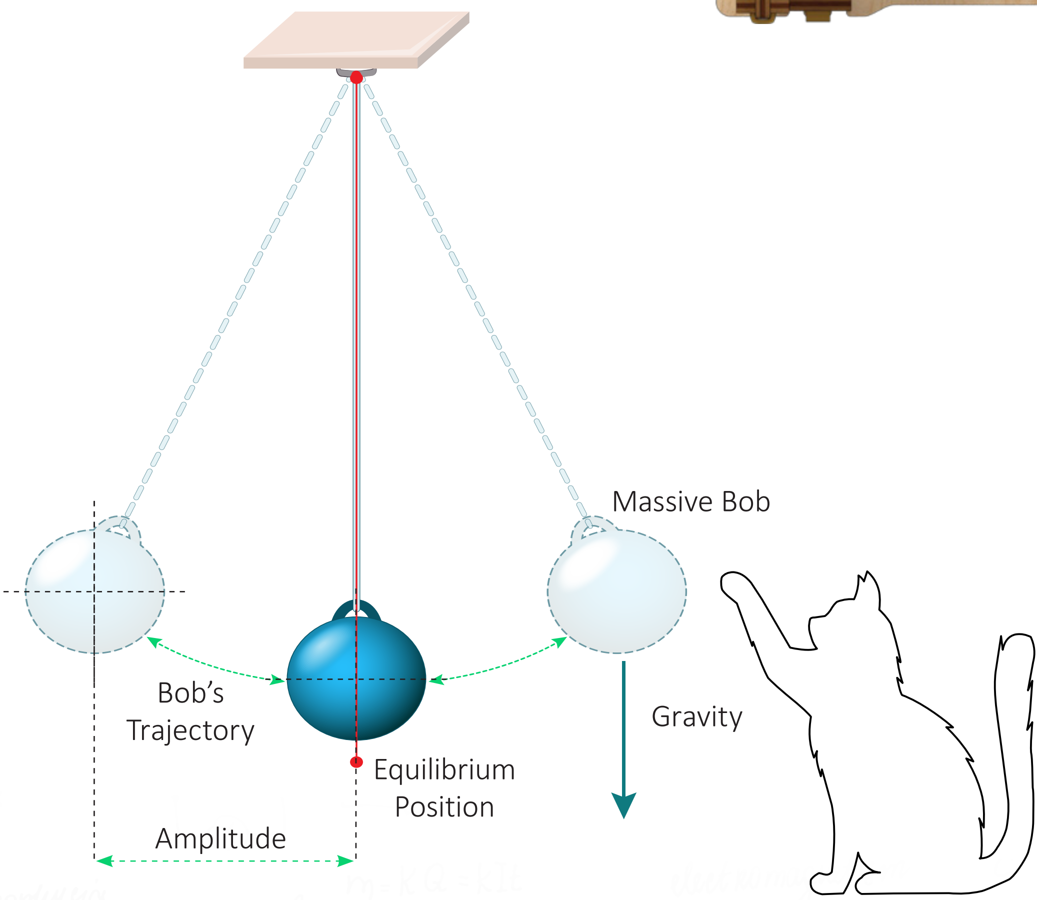
Oscillations are the type of motion in a regular time intervals.

Oscillations can be mechanical, electromagnetic, chemical, thermodynamic, etc. Found in variety of different scientific disciplines, they have a lot in common and dented by the same equations.

In order for an object to oscillate it must be displaced from its equilibrium position.

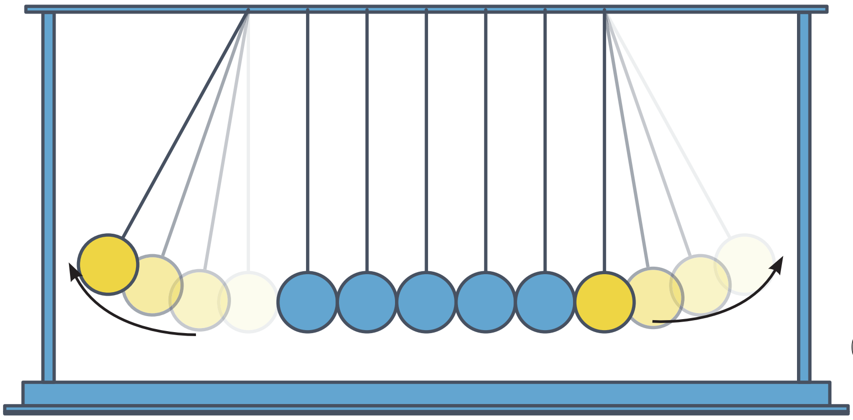
Let’s have a look at the main types of oscillations:

Forced oscillations are oscillations sustained by an external periodic force that compensates for the energy loss in system due to tension. The periodic external force is called a “driving” force.

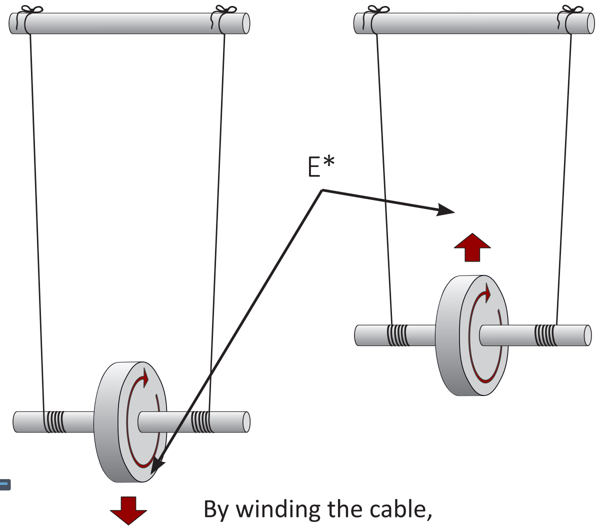


The cat makes the pendulum “oscillate”

Free oscillations are the oscillations of an object under no external influence other than the impulse that initiated the motion.

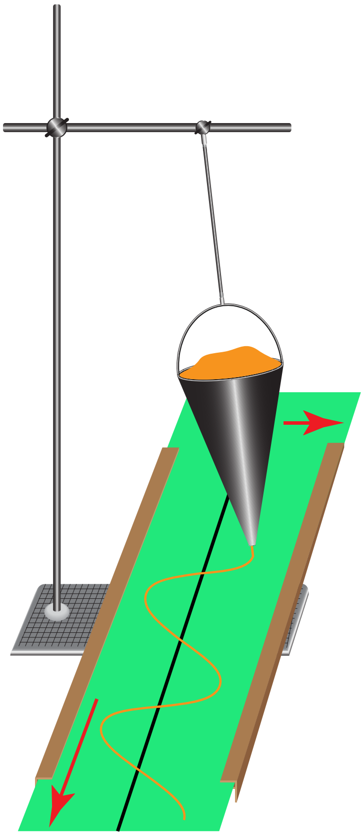


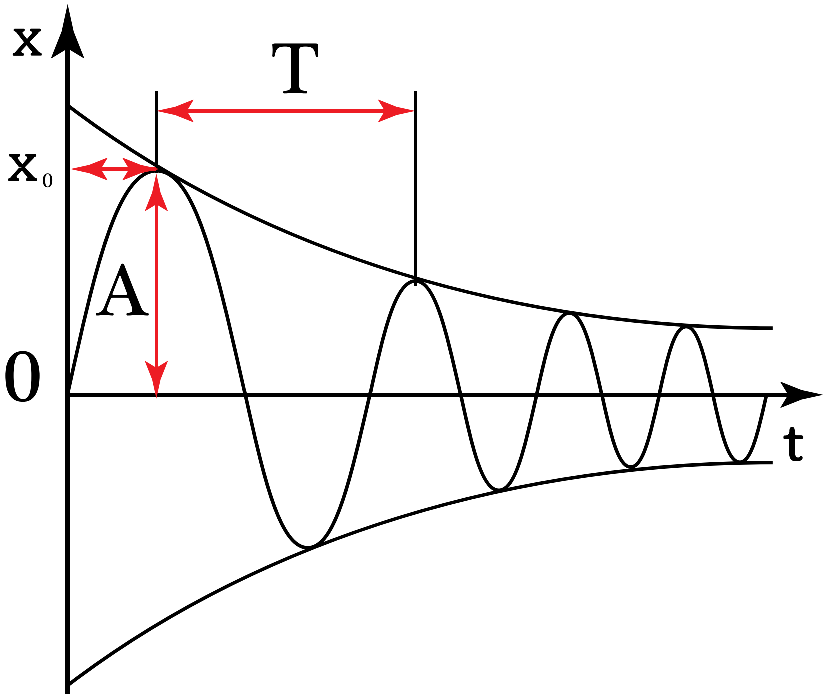
Deviating, the ball accumulates energy



By winding the cable, the pendulum disk accumulates energy.

Damped oscillations are the oscillations that eventually lose energy and their amplitude gradually shrinks. The damping of free mechanical oscillations takes place with decreasing mechanical energy due to resistive force and tension.

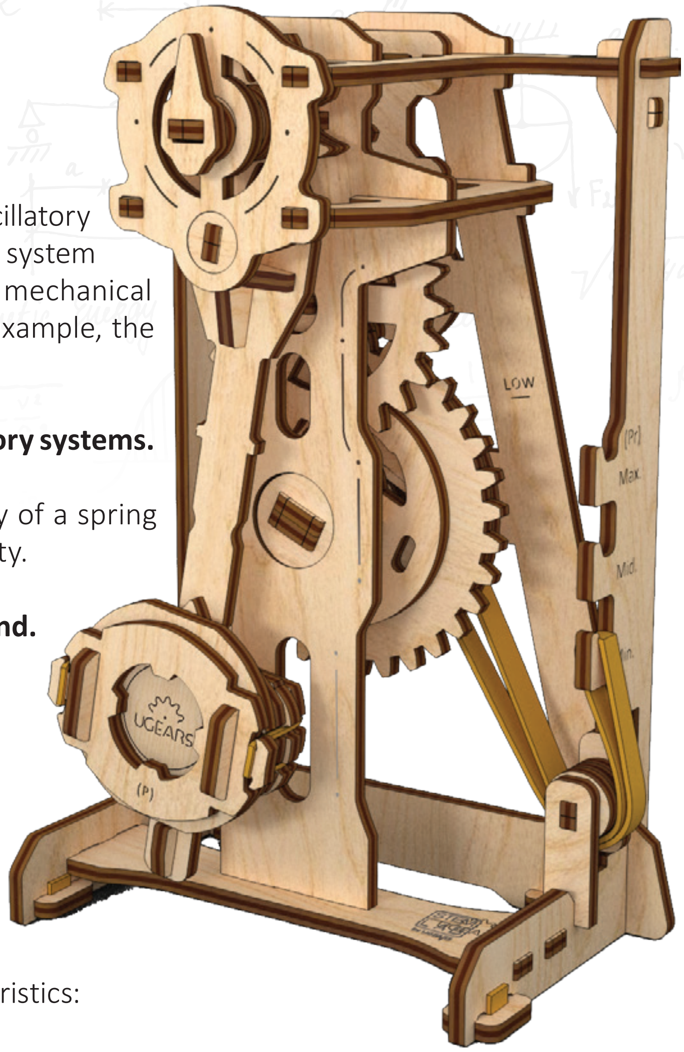




Where:
A- amplitude of oscillations.
T- period of oscillations
t- time scale
x_0- amplitude value

*E – accumulated energy

Self-oscillations occur without outside force but due to the system's own ability to generate and regulate the energy incoming from a permanent source.



A self-oscillatory system has three main elements: oscillatory force, energy source, and a feedback device between the system and the source. The self-oscillatory system can be any mechanical system that can produce damped self-oscillations (for example, the pendulum of a wall clock).

The model you've assembled belongs to self-oscillatory systems.

This type of a system can be powered by the energy of a spring or a potential energy of a weight under the force of gravity.

In our case it is an energy of an extended rubber band.

The oscillations are characterized by the following terms and concepts:

One complete oscillation is considered fulfilled when the oscillating body returned to the starting position and began a new oscillation in the same direction. Oscillatory movements are repetitive and has the following characteristics:

1. Period of oscillations
2. Frequency

Period of oscillations is a time in which the oscillating object makes one full oscillation.
The period of oscillations is denoted as "T" and measures in seconds..
It can be calculated as

$$T = \frac{t}{N}$$

In which
t — is the time the motion lasted
N — is the number of oscillations

Frequency of oscillations is a number of full oscillations in one second.
The frequency is measured in Hertz (Hz) and is denoted as a Greek letter ν.
The frequency can be calculated using equation below:

$$\nu = \frac{N}{t}$$

In which
N — is a number of oscillations
t — is the time the movement lasted
Hz is one oscillation per second. The rate of the human heart on average, has about the same frequency.
The word "herz" in German means "heart".
The period and frequency of oscillation are inversely related:

$$T = \frac{1}{\nu}$$

The unit is named after a famous German physicist Heinrich Hertz (1857 - 1894).

Main energy types in oscillation:

Amplitude - A - the size oscillations, the largest displacement of the oscillating object from its equilibrium position. See illustration below – “Conservation of energy in Pendulum

Energy – E - is one of the main characteristics of matter is a measurement of its movement as well as its ability to produce work.
A body in any condition can have several types of energy, including heat, mechanical, electric, chemical, nuclear as well as potential energy of different physical fields (gravitational, magnet, or electric one). The total of all types of energy a body possesses is its total energy.

Kinetic energy – E(k) – is the **energy** of a moving object. For an object in a state of rest **kinetic energy** equals zero. The **kinetic energy (KE)** of an object depends on the mass of the object (m) and its speed (v).

Potential energy – E(p) – is the **energy** occurred in an object due to its position to other objects or parts of the same object. This means, that is for example the object falling from a certain point to the ground is able to produce work.
Potential energy depends on the heights from which the object falls in relation to the plane towards which it’s falling.

The total mechanical energy is the total of potential and kinetic energies.

$$E=E_{(k)}+E_{(p)}$$

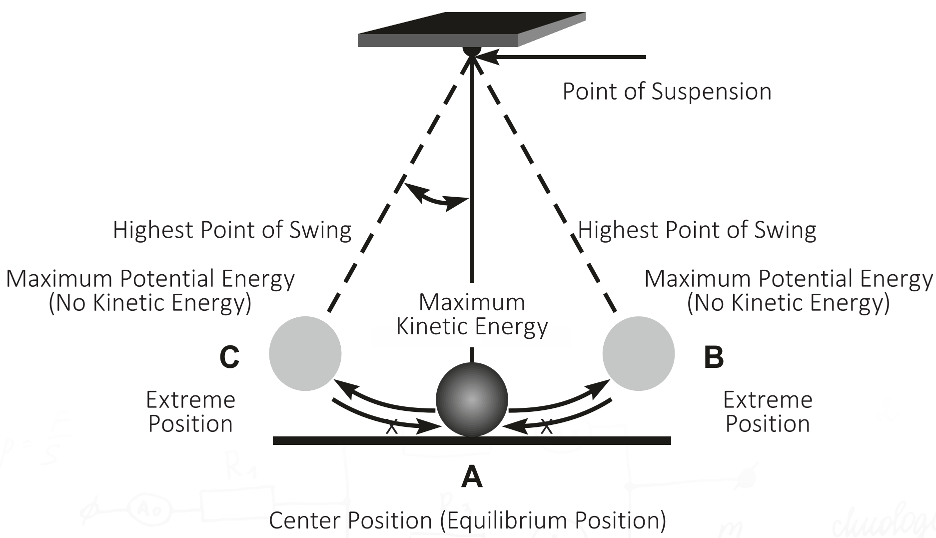
Conservation of energy in oscillatory motion

The oscillations of a pendulum occur due to an initial impulse: the mechanical energy assumed by displacing the weight suspended from the pivot from its equilibrium position.

During the oscillations,

- the pendulum’s maximum speed and kinetic energy occurs when it goes through its equilibrium point.
- Its potential energy is at its maximum when the kinetic energy (speed) equals zero.

When the weight moves from the equilibrium position to the point of maximum displacement (points C and B), kinetic energy transforms into potential energy. When the weight is released and strives to return to the position of equilibrium, potential energy transforms back into kinetic energy.



Conservation of energy in Pendulum

Displacement (x – moving of the oscillating object from its equilibrium position in a current period of time [m].

§5 Technical design and principle of working

THE PRINCIPLE OF WORKING

A good example of a self-oscillating system is an anchor escapement mechanism (Pic. Anchor escapement mechanism with pendulum).

The escape wheel with inclined cogs (A) is attached to the sprocket wheel (B) that transmits impulse from the rubber band (C). The top of the pendulum is made in a form of an anchor (D).

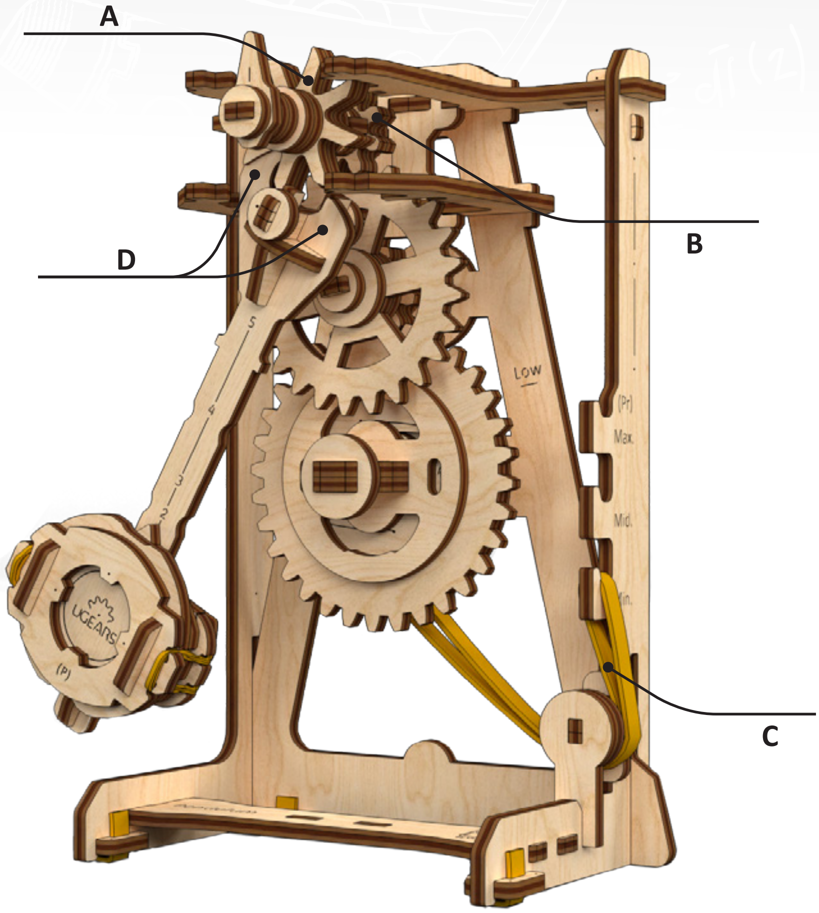
Wall clocks, grandfather clock, tower clock or a watch use a weight as a source of energy. The hand or pocket watch may use a spring to power up and a balance wheel (E) attached to a spiral spring instead of a pendulum. The balance wheel produces rotating oscillations around its radial axis.

The oscillating system in a clock or a watch is a pendulum or a balance wheel. The power source is a weight or a spring. Anchor meshing with escape wheel works as a feedback device. It allows the escape wheel to turn by one cog in one half-a-period.

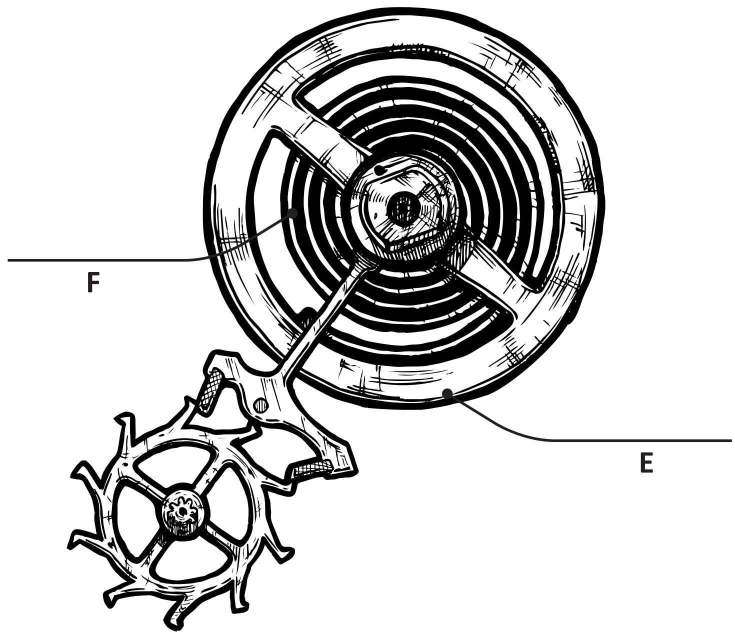
Feedback is produced by the anchor meshing with escape.

It allows the escape wheel to turn by one cog in one half-a-period. Each half-period of the pendulum oscillation, the cog of an escape wheel pushes the anchor in the same direction while transferring a certain energy impulse that compensates for tension energy loss. Thus, the potential energy of a weight or a spring, gradually in small impulses, transfers to the pendulum.

Mechanical self-oscillating systems are widely used and are found virtually everywhere in our daily lives. The examples are many: steam turbines, combustion engines, electric doorbells, the strings of musical instruments as well as wind instruments, and our vocal cords when we talk or sing.



Anchor escapement mechanism with pendulum

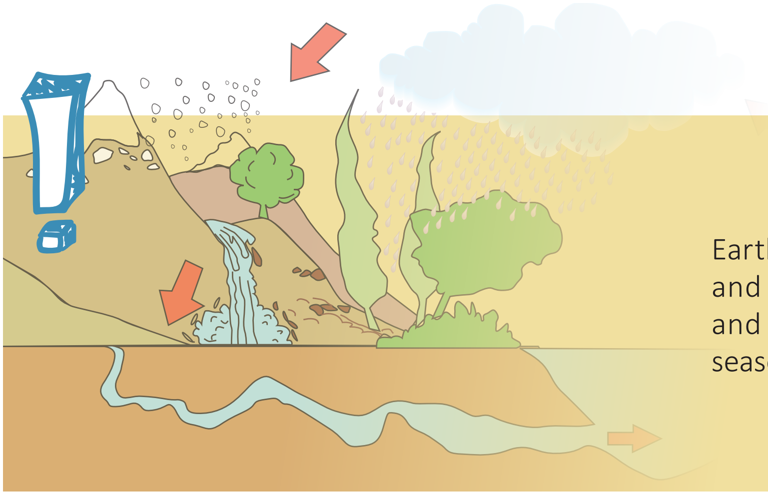


Pic 4. Clockwork with pendulum

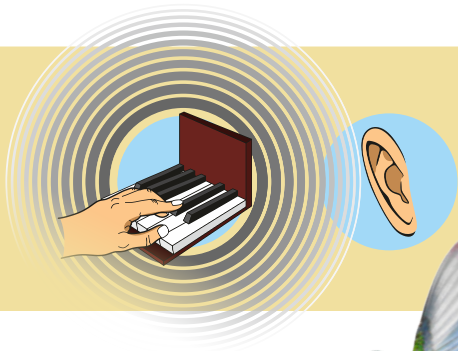
CURIOUS FACTS

Oscillations are one of the most common process in nature and technology.

You find it in the movements of butterflies and birds' wings, tall buildings and high voltage cables in the wind, the pendulum of a clock and a cars' spring suspension, the water levels in a river, and in human body temperature while fighting infection.

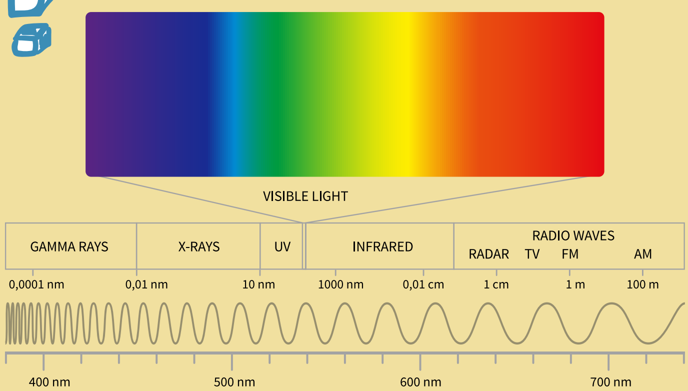


Earthquake ground motion, high and low tide, heart rate, sleep and waking patterns, changing seasons, and many more.



Sound is the vibration of air density and pressure, radio waves are periodic alterations of electric and magnetic field intensity. Visible light is also a type of electromagnetic oscillation, but with different wave-length and frequency.

VISIBLE SPECTRUM



Even our daily work-home commute falls under the definition of oscillations as a process repeating itself in equal periods of time.

A special branch of physics "Theory of oscillations" studies these phenomena. This knowledge is absolutely essential for people involved in plane and shipbuilding, industrial and transport developers, radio and acoustic devices manufacturers.

§6 Formative hands-on tasks

Make you own lab research and study how the period and frequency of pendulum depend on the position of the weight.

Learn what amplitude is. Measuring amplitude, frequency, and period of a pendulum. The dependency of the period of oscillation on the position of the weight.

The completed Pendulum

Objectives: to study how the period depends on the position of the weight. Learn to calculate amplitude using different formulas. Develop logic, science skills, and spatial thinking

Equipment: the Pendulum, a stop-watch, notepad and pen.

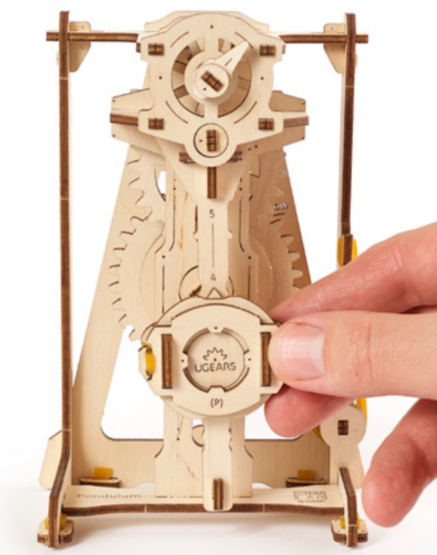
Theoretical background of the experiment:

Assemble the Pendulum and set it on a flat surface. Displace the pendulum from its equilibrium position. Measure the time and count the number of oscillations. In order to ensure the precision of your experiment, repeat your calculations several times. Find the average time tc. The period of oscillations can be found using the following formula:

$$T_c = \frac{t_c}{N} \quad (1)$$

Preparation for the experiment:

Set the Pendulum on a flat surface. We fix the position of the weight. Set the rubber band in the «min» position, put the pendulum weight (bob) in the lower position (if you wish, you can put a coin in the pendulum's bob)



WORK PROGRESS:

***Task 1.* Finding the period of oscillation.**

1. Displace the pendulum from its equilibrium position by 2-3 cm. Note how much time the pendulum takes to make 10 full oscillations.
2. Repeat your experiment 2-3 times with the same settings.
3. Calculate the average time that pendulum uses to complete 10 full oscillations.

$$T_c = \frac{t_1 + t_2 + t_3}{3}$$

4. Use the formula (1) to find the pendulum's period.

***Task 2.* Finding how the position of the weight changes the frequency.**

1. Select one of the positions and anchor the weight in it. Calculate the number of full oscillations in 10 seconds.
2. Calculate the frequency using the following formula:

$$\nu = \frac{N}{t}$$

3. Repeat the experiment several times changing the position of the weight every time.
4. Compare your results.

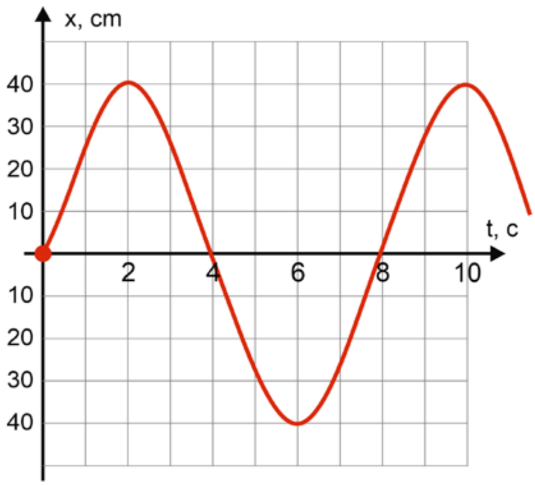
Task 3. **Repeat the experiment described in Task 2 changing the weight of the bob.**

The Pendulum model is a model of an anchor escapement mechanism. The amplitude of the pendulum is permanent and doesn't depend on the tension of the rubber-band or a position of the weight. Only the frequency and the period change

Task 4. **Find the amplitude and period from the graph in the picture.**

Select the correct answers from the following:

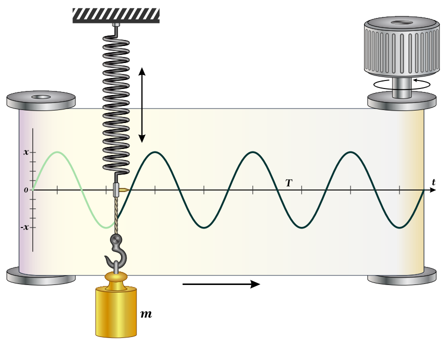
☐ 1. 10 cm
☐ 2. 20 cm
☐ 3. 40 cm
☐ 4. 2 s
☐ 5. 4 s
☐ 6. 6 s
☐ 7. 8 s
☐ 8. 10 s



Task 5. **How will the period of the spring-actuated pendulum change with the weight increasing times 4?**

Select the correct answers from the following:

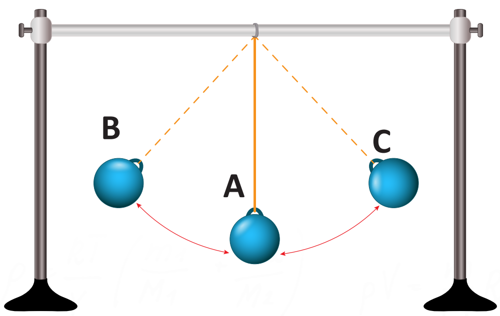
☐ 1. It will increase times 2
☐ 2. It will decrease times 2
☐ 3. It will increase times 4
☐ 4. It will decrease times 4
☐ 5. It will increase times 16
☐ 6. It will decrease times 16



Task 6. **At which point is the potential energy of a pendulum is the highest?**

Select the correct answers from the following:

☐ 1. AC
☐ 2. AB
☐ 3. ABC
☐ 4. BC



CONCLUSIONS:

During the experiment we learnt to measure the amplitude, period and frequency of the pendulum's oscillations.

The following was determined:

- the period and frequency do not depend on the amplitude.
- the period and frequency do not depend on the weight of the bob.
- the period and frequency depend on the position of the bob.

Assessment Task

1. The oscillations of a pendulum are caused by…
☐ a) gravity and elastic force
☐ b) tension and resistance forces
☐ c) gravity and tension.

2. To ensure that minimal convergence of oscillations this value has to be minimal:
☐ a) tension
☐ b) gravity
☐ c) elastic force

3. In the pendulum equilibrium point this value is at its maximum:
☐ a) speed
☐ b) amplitude
☐ c) mass

4. The reason for convergence of oscillations is:
☐ a) tension
☐ b) gravity
☐ c) current

5. Maximum displacement from the equilibrium point is known as…
☐ a) amplitude
☐ b) period
☐ c) oscillation frequency

6. At which point is the potential energy of the pendulum at its maximum?
☐ a) at the point of maximal displacement
☐ b) at the point of equilibrium;
☐ c) the same at every point.

7. The time in which pendulum makes one full oscillation is called:
☐ a) period
☐ b) frequency
☐ c) amplitude

8. Mechanical waves are…
☐ a) the oscillations in linear medium
☐ b) a pendulum's oscillations
☐ c) a repetitive process

9. Pendulum oscillation period depends on…
☐ a) the length of the pendulum
☐ b) the amount of weight
☐ c) the oscillation frequency

10. Who was the first to experiment with the pendulum?
☐ a) Galileo Galilei
☐ b) Christiaan Huygens
☐ c) Isaac Newton

Congratulations! You made it!
Thank you for being with us in this adventure, we hope you had fun and learned a thing or two!